Parabolic Orbits

For the case in which $e = 1$, the integral $J(\theta)$ can be evaluated directly using the identity $\cos\theta = 2\cos^2(\theta/2) - 1$. The result is

$$J(\theta) = (1/4)\ \tan(\theta/2) - (1/20)\tan^5(\theta/2) + C \tag{36}$$

where C is again an arbitrary constant. Since this expression is only defined on the region $r(\theta) > 0$ (i.e., $\cos\theta > -1$), it has no singularities.

Conclusion

The optimal rendezvous equations for a spacecraft near a noncircular Keplerian orbit can be put in a form that avoids the computationally unstable removable singularities found in some earlier papers and the restriction to elliptical orbits found in others. These equations are identical for all noncircular orbits except in the evaluation of an integral whose form is determined by the type of orbit.

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Optimal Aeroassisted Orbital Plane Change with Heating-Rate Constraint

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Introduction

SINCE the pioneering work by London[1] on the use of aerodynamic forces to assist in the orbital plane change, some classical analyses have done by Dickmanns,[2] Vinh,[3] Vinh and Hanson,[4] Walberg,[5] and Miele and Vekataraman.[6] Furthermore, in the recent paper by Miele et al.,[7] several problems have been solved. The sequential gradient-restoration algorithm is used and the purpose of each problem is to minimize, for example, the energy required for orbital transfer, peak dynamic pressure, or peak heating rate, with a prescribed atmospheric plane change. The purpose of this Note is to investigate the optimal trajectories for aeroassisted orbital plane change subject to a heating-rate constraint when the plane-change angle is being maximized.

In this Note, we use the modified Chapman variables[3] to derive the set of dimensionless equations of motion and the dimensionless heating rate. Hence, it suffices to just specify the maximum lift-to-drag ratio $(L/D)_{max}$ as the sole physical characteristic of the vehicle. As a result of the variational formulation, we have a two-point boundary-value problem (TPBVP) in which the constraint forms an interior boundary condition. In order to circumvent the difficulties in numerical computation, the multiple shooting method and the continuation method[8] are used. The modified Newton method is used to induce and accelerate the convergence.

Dimensionless Equations of Motion and Heating Rate

The motion of the re-entry vehicle, considered as a mass point, is defined by the six variables r (range), θ (longitude), ϕ (latitude), V (velocity), γ (flight-path angle), and ψ (heading angle) as shown in Ref. 3.

Using a parabolic drag polar of the form

$$C_D = C_{DO} + KC_L^2 \tag{1}$$

we define the normalized lift coefficient

$$\lambda = C_L/C_L^* \tag{2}$$

where C_L^* is the lift coefficient corresponding to the maximum lift-to-drag ratio E^*. With given values of C_{DO} and K assumed constant at hypersonic speed, we can easily compute the values of C_L^*, C_D^*, and E^*. The Earth is assumed at rest and with a locally exponential atmosphere,

$$\mathrm{d}\rho/\rho = -\beta \mathrm{d}r \tag{3}$$

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where β, the reciprocal of the scale height, is a function of the radial distance r. Its gravitational force field is the usual inverse-square force field. Without loss of generality, we can use the equatorial plane as the reference plane. By introducing the modified Chapman variables Z and v, and the dimensionless arc length s,

$$Z = (\rho S C_L^*/2m)\sqrt{(r/\beta)}, \qquad v = V^2/gr$$
$$s = \textstyle\int_0^t (V/r)\cos\gamma \, dt \tag{4}$$

we have the dimensionless equations of motions[3]

$$dZ/ds = -k^2 Z \tan\gamma \tag{5a}$$

$$dv/ds = -kZv(1+\lambda^2)/E^* \cos\gamma - (2-v)\tan\gamma \tag{5b}$$

$$d\gamma/ds = kZ\lambda \cos\sigma/\cos\gamma + (1 - 1/v) \tag{5c}$$

$$d\theta/ds = \cos\psi/\cos\phi \tag{5d}$$

$$d\phi/ds = \sin\psi \tag{5e}$$

$$d\psi/ds = kZ\lambda \sin\sigma/\cos^2\gamma - \cos\psi \tan\phi \tag{5f}$$

The aerodynamic controls are in the form of the bank angle σ and the normalized lift coefficient λ. Equations (5) contain only two constant parameters that need to be specified. The first parameter, $k^2 = \beta r$, is a characteristic of the atmosphere; and the second parameter, E^*, is a performance characteristic of the vehicle. For the present numerical example, we shall take the values $k^2 = 900$ for the Earth and $E^* = 1.5$ for the vehicle.

It is required to constrain the peak value of the heating rate at a particular point of the spacecraft, for instance, the stagnation point. The dimensionless heating rate of the stagnation point can be constructed as[9]

$$\bar{q} = \sqrt{k} Z^{1/2} v^{3/2} \tag{6}$$

and the heating-rate constraint can be written in the form

$$S(Z, v) = \sqrt{k} Z^{1/2} v^{2/3} - \bar{q}_{\max} \leq 0 \tag{7}$$

where S is an inequality constraint on a function of the state variables Z and v, and $\bar{q}_{\max}$ is the maximum heating rate specified. By taking the total derivative of Eq. (7) with respect to s and substituting dZ/ds and dv/ds from Eqs. (5), we have

$$S' = -\bar{q}_{\max}\{[k^2/2 + 3(1/v - 1/2)] \tan\gamma + 3kZ(1+\lambda^2)/2E^* \cos\gamma\} = 0 \tag{8}$$

where S' represents the first total derivative of S with respect to s. Because S' is explicitly dependent on one of the control variables λ, S is a first-order state variable inequality constraint.

Variational Formulation

Using the maximum principle, we introduce the adjoint vector p to form the Hamiltonian[10]

$$H = -k^2 Z p_z \tan\gamma - p_v[kZv(1+\lambda^2)/E^* \cos\gamma + (2-v)\tan\gamma] + p_\gamma[kZ\lambda\cos\sigma/\cos\gamma + (1-1/v)] + p_\theta \cos\psi/\cos\phi + p_\phi \sin\psi + p_\psi[kZ\lambda \sin\sigma/\cos^2\gamma - \cos\psi\tan\phi] + \mu S' \tag{9}$$

where μ is the Lagrange multiplier and is a function of s, and we have $\mu \leq 0$, when $S = S' = 0$, or $\mu = 0$, when $S < 0$. The optimal bank control can be derived from $\partial H/\partial\sigma = 0$ and it gives

$$\tan\sigma = p_\psi/p_\gamma \cos\gamma \tag{10}$$

For the optimal lift control, we have $\partial H/\partial\lambda = 0$. Therefore, when the trajectory is not on the heating-rate constraint (i.e., $S < 0, \mu = 0$),

$$\lambda = (E^*/2vp_v \cos\gamma)[(p_\gamma \cos\gamma)^2 + p_\psi^2]^{1/2} \tag{11}$$

and when the trajectory is on the constraint (i.e., $S = S' = 0$, $\mu \leq 0$),

$$\lambda = \{-(E^* \sin\gamma/kZv)[(k^2/3 - 1)v + 2] - 1\}^{1/2} \tag{12}$$

with

$$\mu = -(E^*/3\lambda\bar{q}_{\max})\{2\lambda v p_v/E^* - (1/\cos\gamma)[(p_\gamma\cos\gamma)^2 + p_\psi^2]^{1/2}\} \tag{13}$$

It has been shown that the system has four integrals

$$H = C_0, \qquad p_\theta = C_1, \qquad p_\phi = C_2 \sin\theta - C_3\cos\theta$$
$$p_\psi = C_1 \sin\phi + (C_2\cos\theta + C_3\sin\theta)\cos\phi \tag{14}$$

where C_i are constants of integration. For the remaining three adjoint variables p_z, p_v, and p_γ, we define the following three new adjoint variables

$$p = k^2 Z p_z, \qquad N = v p_v, \qquad Q = p_\gamma \cos\gamma \tag{15}$$

In terms of these variables, the Hamiltonian integral becomes

$$H = -kNZ(1+\lambda^2)/E^* \cos\gamma + KZ\lambda(Q\cos\sigma + p_\psi \sin\sigma)/\cos^2\gamma - Q(1-v)/v\cos\gamma - [vP + (2-v)N]\tan\gamma/v + H_1 + \mu S' \tag{16}$$

where

$$H_1 = C_1\cos\phi\cos\psi - C_2(\sin\phi\cos\theta\cos\psi - \sin\theta\sin\psi) - C_3(\sin\phi\sin\theta\cos\psi + \sin\theta\sin\psi) \tag{17}$$

is a function of the state variables. The new adjoint variables P, N, and Q, are governed by the system of differential equations

$$dP/ds = -k^2\{-H_1 + Q(1-v)/v\cos\gamma + [vP + (2-v)N]\tan\gamma/v\} + (3/2)\mu\bar{q}_{\max} k^3 Z(1+\lambda^2)/E^* \cos\gamma \tag{18a}$$

$$dN/ds = -2N\tan\gamma/v - Q/v\cos\gamma - 3\mu\bar{q}_{\max}\tan\gamma/v \tag{18b}$$

$$dQ/ds = 2H_1\sin\gamma + Q(1-1/v)\tan\gamma + (1 - 2\sin^2\gamma)[vP + (2-v)N]/v\cos\gamma - kNZ(1+\lambda^2)\tan\gamma/E^* + (1/2)\mu\bar{q}_{\max}\{E^*[k^2 + 3(2/v - 1)] + 3kZ(1+\lambda^2)\sin\gamma\}/E^*\cos\gamma \tag{18c}$$

It is proposed, for a prescribed speed depletion, to maximize the plane change i, that is, the function

$$J = -\cos i_f = -\cos\phi_f \cos\psi_f \tag{19}$$

We have the transversality conditions

$$p_{\phi f} = \partial J/\partial\phi_f = \sin\phi_f \cos\psi_f \tag{20a}$$

$$p_{\psi f} = \partial J/\partial\psi_f = \cos\phi \sin\psi_f \tag{20b}$$

Let s_1 be the point of entering the heating constraint boundary and s_2 be the point of leaving it. The condition $S(Z,v) = 0$ forms an interior boundary condition. Since it is a function of Z and v, and the independent variable s does not

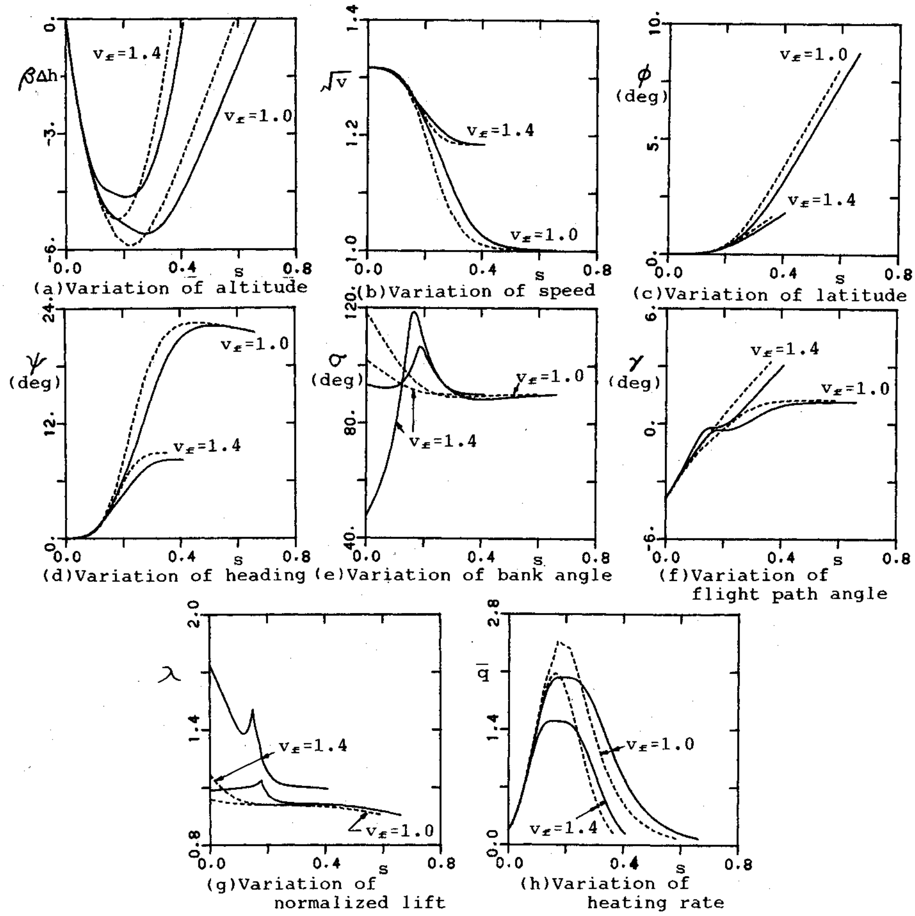

Fig. 1 Optimal trajectories: the unconstrained trajectories (dashed line) and the heating-rate-constrained trajectories (solid line).

appear explicitly, we have the following conditions:

$$p_z(s_1-) = p_z(s_1^+) + \pi(\partial S/\partial Z) \qquad \text{at } s = s_1 \qquad (21a)$$

$$p_v(s_1-) = p_v(s_1^+) + \pi(\partial S/\partial v) \qquad \text{at } s = s_1 \qquad (21b)$$

and

$$H(s_1^-) = H(s_1^+) \qquad (21c)$$

where we let s_1^- and s_2^- signify just before s_1 and s_2, respectively, and s_1^+ and s_2^+ signify just after s_1 and s_2, respectively. The π is a constant Lagrange multiplier to be determined so that the condition $S(Z,v) = 0$ is satisfied at $s = s_1$. This means that we pick the entry point as the place to satisfy the interior boundary condition, and thus p_z and p_v are discontinuous at this point. The condition $\mu = 0$ is used to determine the exit point from the boundary, since we assume that control variables do not have constraints and they are continuous at the exit point.

Numerical Results

By the variational formulation, the problem is transformed into a well-defined TPBVP. Because of the high nonlinearity of Eqs. (5) and (18) and the high sensitivity of the initial guess, the continuation method and the multiple shooting technique[8] are used to overcome these difficulties. The maneuver considered involves the noncoplanar transfer from a high Earth orbit (HEO) to a low Earth orbit (LEO). The HEO is the geosynchronous Earth orbit (GEO) and the LEO is the outer boundary of the Earth's atmosphere. We arbitrarily choose a small entry flight-path angle $\gamma_e = -3.9$ deg. From Ref. 4, the entry speed is calculated as $v_e = 1.7327$. We can use the relation $\beta\Delta h = \ell n(Z_e/Z)$ to transfer the dimensionless altitude Z into the actual altitude difference. One unit of $\beta\Delta h$ equals 7.162 km. Thus, the initial and final conditions are

$$s_0 = 0$$

$$(Z_0, v_0, \gamma_0, \theta_0, \phi_0, \psi_0) = (0.0002,\ 1.7327,\ -3.9 \text{ deg},\ 0 \text{ deg},\ 0 \text{ deg},\ 0 \text{ deg})$$

$$s_f = \text{free}$$

$$(Z_f,\ v_f,\ \gamma_f,\ \theta_f,\ \phi_f,\ \psi_f) = (0.0002,\ \text{prescribed, free, free, free, free})$$

Since the final arc length s_f, final flight-path angle γ_f, and final longitude θ_f are free, we have

$$p\gamma_f = 0, \qquad H = C_o = 0, \qquad p_\theta = C_1 = 0 \qquad (22)$$

Figure 1 presents the optimal trajectories for the cases when the exit speeds v_f are 1.4 and 1.0, respectively. For comparison, both unconstrained and constrained trajectories are plotted. For supercircular speed exit ($v_f = 1.4$), the dimensionless heating rate constraint is $\bar{q}_{\max} = 1.5$, and for circular speed exit ($v_f = 1.0$), it is $\bar{q}_{\max} = 2.0$. From the figure, we can see that the heating rate stays on the boundary $\bar{q}_{\max}$ for an obvious duration of time. In Fig. 1a, it can be seen that the heating-rate-constrained optimal trajectories remain longer in the denser atmosphere, but the altitude drop is smaller than for the unconstrained trajectories. The variations on the bank angle and the normalized lift coefficient for the constrained trajectories are much more pronounced. In summary, during the early phase of the aeroassisted flight, the vehicle has high lift coefficient (as shown in Fig. 1g) and small bank angle. This means that the vehicle turns and descends more slowly during this phase in order to get onto the constraint boundary smoothly. As we expected, the trajectories stay on the constraint boundary when the flight is at the bottom of the trajectories. The vehicle maneuvers with high lift coefficient and large bank angle to obtain the maximum rate for turning. The maximum orbital plane-change angle for the heating-rate-constrained case with $v_f = 1.4$ is 8.5393 deg (23.0937 deg with $v_f = 1.0$), and for the unconstrained case it is 9.0851 deg (23.2636 deg). Hence, the penalty due to the constraint is 7.38% (0.74%). The normalized lift coefficient is discontinuous at s_1 and has a value 1.4474 at s_1^- and 1.4267 at s_1^+ for supercircular speed exit.

Conclusions

This paper presents the exact solution for the heating-rate-constrained optimal trajectory that maximizes the orbital plane-change angle by using the aerodynamic forces. With the variational formulation, a two-point boundary-value problem is formulated in which the constraint forms an interior boundary condition. The multiple shooting method and the continuation method are used for the numerical computation, and the modified Newton method is used to induce and accelerate convergence. The optimal trajectories with the initial entry speed corresponding to entry speed for a direct return from a geosynchronous Earth orbit and specified final exit speed are solved. From the results, it can be seen that the constrained trajectory has a large deviation from the unconstrained one. It stays much longer in the atmosphere, but the penetration into the atmosphere is more shallow. The trajectory enters the constraint boundary at a certain point of the descending arc and then leaves the boundary after a certain period of time when the vehicle is in its ascending arc. The vehicle performs pronounced maneuvers to obtain the maximum rate for turning during this period of time. For a shallow entry with 10% of speed depletion, the penalty on the orbital plane-change angle due to the heating-rate constraint is 7.38%.

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On the Level 2 Ratings of the Cooper-Harper Scale

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Introduction

THE acceptance and application of the Cooper-Harper flying qualities rating scale are universal. To attempt any "improvement," therefore, would appear to be folly. In their latest paper,[1] Cooper and Harper discuss potential problems with definitions in the level 2 region (see Fig. 1). As is stated in Ref. 1, in that region there "are adjectives to classify the severity of the deficiencies which warrant improvement. It is possible that the choice of one of these descriptors could conflict with the performance/compensation selections. This has not appeared to be a problem but, if it should be, the pilot should note the conflict and the reason for his final selection." This potential conflict between descriptors of performance and workload is also likely to yield different interpretations among different pilots. We might expect, therefore, more scatter in pilot ratings the more pilots are used, and especially if pilots of different backgrounds are used.

The potential conflict noted by Cooper and Harper was expected in a simulation performed in support of an aircraft development program with both performance and workload requirements. The environmental conditions specified for the landing performance meant that ratings in the 4–6 region were expected. In addition, evaluations were to be performed with pilots of different backgrounds. This author took the approach, therefore, of providing a more explicit definition of parts of the Cooper-Harper decision tree and rating descriptors. The intent is not to change any of the scale, but to amplify the definitions and remove ambiguities. The objective of this Note is to present these expanded definitions.

The Decision Tree

Figure 2 shows the decision tree with a different wording for the last decision. Cooper and Harper wrote, "Is it satisfactory without improvement?" This is equivalent to asking "Is desired performance attainable with a tolerable workload?" The latter wording is more consistent with the basic theme of assessing workload and performance; it is also believed to be less ambiguous for the non-test-pilot. The line pilot is more usually faced with flying an aircraft that is "satisfactory" by definition, whatever the flying qualities.

Note that the suggested wording does not change the level 1 ratings. It does, however, allow cleaner bounds on the different levels to be inferred. Level 1 means that desired performance can be achieved with acceptable workload. Level 2 means that desired performance can be achieved but workload is unacceptable. Also, adequate performance can be attained with acceptable workload. Level 3 means that desired performance cannot be achieved, and adequate performance may be achieved but workload is unacceptable. Thus, there is a more explicit consideration of both desired and adequate performance in the level 2 and 3 ratings, rather than an indistinct transition from desired to adequate performance.

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